

The End of the Many-Worlds?
(or Could we save Everett's interpretation)

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Abstract

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I. INTRODUCTION

The aim of this text is to discuss in a rather critical way the so called many worlds interpretation (MWI) proposed originally by H. Everett in 1957 [1]. I am a proponent of the pilot wave interpretation (PWI) defined by L. de Broglie in 1927 and D. Bohm in 1952 [2] and while MWI shares with PWI a strong commitment for determinism there are also fundamental differences between the two approaches. The most important one concerns probably how both theories attempt to interpret quantum probability within their own ontological frameworks and this will be the topic of the present chapter. As I will show however MWI is more difficult to accept than PWI in the sense that it has dramatic problems with its ontology which cannot be ignored if we want to interpret probabilities.

First, let us say a few words about ontology. MWI and PWI are realistic approaches to quantum mechanics. This means that they are trying to introduce a clean ontological structure into the formalism of quantum mechanics. What is indeed so extraordinary about quantum mechanics is that the formalism appeared in 1925-1927 without a clear ontological framework in complete opposition with classical approaches. Instead, N. Bohr, W. Heisenberg, M. Born and others managed to develop a pragmatic and instrumentalist interpretation in which only macroscopic apparatuses and detectors possess a ‘clear’ definition. This way of thinking, very much in harmony with the dominant positivism of this time, relies on the need to consider as existing only what is seen by an ‘observer’ (i.e. the so called quantum observable). In classical physics this positivistic approach was already introduced by the philosopher E. Mach, with his phenomenalist philosophy of science, and by the chemist W. Oswald with his strong criticism of atomism (i.e., in his debate with L. Boltzmann). The logical positivism of M. Schlick, R. Carnap, P. Franck, and H. Reichenbach perpetuated this methodology in the XX’s century and considered that ontology is pure metaphysics and should be removed of any positive science (actually they are strongly mistaken: any science is necessarily metaphysical on its theoretical ground as shown already by D. Hume. Theory is a pure creation of the human mind and needs to be tested with experimental facts. This is the basis of the hypothetico-deductive method which was in particular defended by L. Boltzmann and later by A. Einstein in his debated with W. Heisenberg and N. Bohr). However, Bohr’s way of thinking is not completely identical to these various positivist approaches. For Bohr the problem is mainly experimental and is associated with the

existence of the quantum of action \hbar . Indeed, the classical ontologies based on waves and particles are not able to give a clear unambiguous picture of quantum reality as shown for instance by the famous wave-particle duality paradox. Therefore, one should try a different minimalist approach in which these classical concepts, while necessary for the empirical description of phenomena, do not have the same ontological values as in the classical world. The Copenhagen interpretation says something like that: Take the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle_t = \hat{H}|\Psi\rangle_t \quad (1)$$

with its wave function $|\Psi\rangle_t$ and its hamiltonian \hat{H} . Don't try to 'see' a physical propagation of a wavy thing; use it as a practical tool for defining a probability obtained by a 'classical observer'. The observer, sentient or not, (detector or automaton are part of the interpretation as well) possesses a well defined position in space and time and is therefore foreign to the quantum formalism. Quantitative statements are introduced into the theory through Born's probability rule which reads

$$P_\alpha(t) = |\langle\alpha|\Psi_t\rangle|^2 \quad (2)$$

where P_α , the probability of the outcome α (associated with the observable operator \hat{A}), is related to the quantum state by squaring the norm of the amplitude $c_\alpha(t) = \langle\alpha|\Psi_t\rangle$ (where we have the eigenvalue relation $\hat{A}|\alpha\rangle = \alpha|\alpha\rangle$). Ontological questions about what happens to the system between the preparation and the measurement are beyond the scope of Bohr's interpretation. This answer is fine for an experimentalist in his lab and can be used with confidence for all practical purposes (FAPP). However, this methodology keeps open some unsolved fundamental questions. Questions such as the meaning of the wave function for the Universe, or the Schrodinger cat, or Wigner's Friends, or the Heisenberg shifty split, and so on and so forth are very pertinent and they cannot simply be escaped by labeling them metaphysical. After all, the quantum wave theory was developed by Schrodinger before Bohr gave his interpretation and there is no reason why the Copenhagen method should be the only pertinent one. Actually, the situation is even a bit ironical since Bohr's interpretation came only of late. De Broglie proposed his own 'double solution' theory already in 1925, before the Schrodinger equation was discovered (de Broglie was using the Klein-Gordon equation which he invented for his own purpose). Schrodinger proposed his own theory in 1926. In fact de Broglie's view contains as a sub-product the PWI while Schrodinger's, if

correctly interpreted in the configuration space has as a necessary consequence MWI (as is recognized by Everett himself in his PhD thesis [1]).

II. EVERETT AND BOHM

Now, I will try to define briefly the ontology of MWI and PWI. I will start with PWI. In PWI the wave function $|\Psi\rangle_t$ has an ontological meaning independently of the existence or nonexistence of the observer. $|\Psi\rangle_t$ is an actual guiding wave in a real quantum field for particles which are somehow surfing on the wave. Here I will be rather conservative and consider only the non-relativistic case for a single particle of mass m without magnetic potential. The velocity of the point like particle in PWI is given by

$$\mathbf{v}(t) = \frac{d}{dt}\mathbf{x}(t) = \frac{\hbar}{m}\text{Im}\left[\frac{\nabla\psi(\mathbf{x}(t), t)}{\psi(\mathbf{x}(t), t)}\right]. \quad (3)$$

This guidance formula is enough for solving paradoxes such as wave-particle duality by defining a complex trajectory for the particle. The wave carries the particle into regions where the field is non vanishing and omits regions where the quantum field cancels, therefore offering a contrasting view to the Bohr-Heisenberg dictum that such kind of representation should be prohibited (in the same way the famous ‘no-go’ theorem by von Neumann [3] vanishes into pure smoke [2, 4]). Alternatively, we can introduce a quantum potential Q_Ψ acting on the particle and modifying the newtonian force induced by the external potentials $V(\mathbf{x}, t)$ [2]. In the configuration space for many particles $\mathbf{x}_1(t), \mathbf{x}_2(t)$ etc... the theory is highly nonlocal and can be used to solve the Einstein Podolsky Rosen paradox (EPR)[5] in full agreement with Bell’s theorem [2, 6]. I point out that I was deliberately quite unprecise and vague concerning the nature of the quantum field Ψ . Indeed, in PWI, Ψ guides the particles but there is no reaction of the particle on the wave. It is for this reason better to wait for a better understanding of the ontology of the wave function in the future. Probably, PWI, if it survives, will have to be modified or completed by a more satisfying theory in which particles and fields (manifested through waves) will be dynamically connected. This was the hope of both de Broglie and Bohm with different strategies and we should not here comment further. Anyway, even if such a theory would exist one day, this doesn’t mean that the current PWI will not be correct anymore in the same way that Newton’s theory of gravitation is not completely invalidated by general relativity. Additionally, as it was

pointed out many times by de Broglie there is a strong analogy between the status of Ψ in the PWI and the action $S(q, t)$ in the old Hamilton-Jacobi theory. Therefore, we will here only consider Ψ as an effective field or a ‘nomological entity’ keeping its understanding for future works.

Now, of course PWI makes sense only if we can introduce probabilistic elements into the theory in order to explain Born’s formula Eq. 2. Schrodinger’s equation contains enough material to do that unambiguously. Indeed, from the local current conservation law $\nabla \cdot \mathbf{J} + \partial_t \rho = 0$ with $\mathbf{J}(\mathbf{x}, t) = \mathbf{v}(t)\rho(\mathbf{x}, t)$ and

$$\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2, \quad (4)$$

it is obviously ‘natural’ to interpret $\rho(\mathbf{x}, t)$ as a density of presence for the particle located at \mathbf{x} at time t . The local conservation plays the role of Liouville theorem’s in statistical mechanics. Therefore, if $\rho(\mathbf{x}, t)$ is effectively interpreted as a density of probability of presence $P(\mathbf{x}, t)$ at a given time the interpretation will still be valid at any other times in agreement with the conservation law. This was essentially the reasoning of de Broglie in 1927 and of Bohm in 1952. However, one can try to go further and attempt to justify the probability law $P(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$ with other assumptions. This will be very similar to modern statistical mechanics trying to justify the microcanonical, or canonical Boltzmann-Gibbs ensemble with deeper reasonings. Several attempts have been done in the recent years focussing on either H-theorem, coarse graining, Boltzmann-Typicality, and deterministic chaos (for a review see [7]). I will not discuss that further since my aim was only to point that PWI is very similar to classical physics, no better no worse. The difficulties and questionings about equilibrium and non-equilibrium cited by Boltzmann are now translated in the PWI framework into the question of how to justify the uniqueness of quantum equilibrium and how to reach such an equilibrium. The solutions are probably very similar for the classical and quantum cases (but the role of entanglement and non locality is still not so clear) so that solving the problem in classical statistical mechanics would give a strong insight on the quantum PWI version.

Let us now turn to MWI. As said before, MWI is mainly the strict application of Schrodinger’s original theory in the configuration space. In MWI, ψ is an ontological field entity as in PWI. But here there is not particle at all only a continuous wave! For Schrodinger in the case of the single electron, the wave function $e|\psi(\mathbf{x}, t)|^2$ describes a local electron

charge density not a probability. The quantum field $\psi(\mathbf{x}, t)$ in such a theory is somehow very similar to the classical electromagnetic field in the old fashioned Maxwell's theory, or to the metric tensor in general relativity. In recent years, practitioners and proponents of MWI, in particular L. Vaidman, often called $|\psi(\mathbf{x}, t)|^2$ a measure of existence or a degree of reality [8] (the definition of L. Vaidman is more general of course but it agrees with mine if by existing quantum 'world' we mean a state like $|\mathbf{x}\rangle$). However since there is no preferred basis in the Hilbert space the choice is arbitrary). This is clearly reminiscent of Schrodinger's language about charge density. Of course, if we think of the electron as a delocalized charge distribution, we get the obvious objections already given by Bohr and Heisenberg that a wave packet is spreading through space and time and that this could not explain the experimental facts where electrons are appearing as localized grains or quanta on detectors. In a diffraction or interference experiment the electron would go through both holes and could not create a singular event on the screen. This is the well known historical reason why the Schrodinger solution was quickly abandoned already in 1927. Furthermore, observe that already for a single electron wave function the spatial coordinate representation doesn't have any particular status in the theory. We could also have chosen a momentum representation $|\mathbf{k}\rangle$ instead of $|\mathbf{x}\rangle$ this would not have modified the information contained into the quantum state. Therefore, there is no privileged basis in the MWI of Schrodinger and Everett in the same way that it is not more or less physical to consider the Fourier transform $\tilde{E}(k)$ of an electromagnetic field as more or less real than the field $E(x)$ itself. However, we have something new in quantum physics. A superposition such as $|\text{here}\rangle + |\text{there}\rangle$ for a single electron means that two wave packets localized in the 3D space are superposed. Since however experiments show that an electron is only detected here OR there, it seems to us that MWI should have fundamental difficulties in dealing with this reality.

However, Everett resurrected Schrodinger's theory by introducing entanglement in the many particles configuration space. His hope was that entanglement added to the Schrodinger idea would solve completely the measurement paradox and all other contradictions. We will see that this is not so easy. To start with entanglement, consider an ideal two-particle Universe. In the coordinate representation and at a given time t , the wave function for the system is written $\psi(\mathbf{x}_1, \mathbf{x}_2, t)$. What is here the meaning of \mathbf{x}_1 , and \mathbf{x}_2 ? For a single electron \mathbf{x} is a spatial coordinate labeling the point in the 3D space. But here we have two points of coordinates \mathbf{x}_1 , and \mathbf{x}_2 . This means that if we want to interpret

$\psi(\mathbf{x}_1, \mathbf{x}_2, t)$ as an ontological field we should extend a bit the classical framework of field theory. Generally, in classical field theory we consider only local fields defined at a single position. The field itself can obey a nonlinear equation in order to create for example solitons or ‘bunched’ field. Here however, the quantum field ψ require two coordinates for its definition \mathbf{x}_1 , and \mathbf{x}_2 . This involves a new form of non-locality or wholeness while the equation (i.e Eq. 1) is kept strictly linear. How to call such a field? Maybe ‘web-field’ could be a good alternative name to wave function since it actually represents a correlation between two or many points in space and time. The theory can of course be extended to the relativistic domain using the Tomonaga-Schwinger equation for the functional $\Psi[\sigma]$ where σ is a space-like hyper-surface in the 4D Universe. Now, how will this solve the paradox of the spreading electron which was a dead-end for the old Schrodinger interpretation? Consider again our wave function $\psi(\mathbf{x}_1, \mathbf{x}_2, t)$. Suppose at the initial time the system factorizes, i.e., $\psi(\mathbf{x}_1, \mathbf{x}_2, t) = \psi_1(\mathbf{x}_1, t)\phi_2(\mathbf{x}_2, t)$. We can take $\psi_1(\mathbf{x}_1, t)$ as a freely expanding wave packet diffracted by an external potential like a double slit screen (the fact that the screen or more generally an external potential is a classical concept is here irrelevant and it is there only to simplify the discussion). The second wave function $\phi_2(\mathbf{x}_2, t)$ is associated with a localized wave-packet confined in, for example, a Coulomb potential, let’s says, in a energy level E_0 . When the first electron collides with the second the system can exchange energy and momentum and this represents some information transfer to be used in a basic measurement protocol. In other words, the modification of the second electron state will lead to entanglement and non-locality. Consider the final state $\sum_i \psi_i(\mathbf{x}_1, t)\phi_i(\mathbf{x}_2, t)$ where the sum is over possible outcomes. If we consider only a two state system for ϕ_i then we have here a basic electron detector plate with the ground state meaning undetected while the excited states are recorded. Of course, we could also introduce a third electron localized in state $\chi_3(\mathbf{x}_3, t)$ before the interaction and actually factorized from the rest of the wave function. After the interaction we will obtain a state like

$$\psi'_1(\mathbf{x}_1, t)\phi_g(\mathbf{x}_2, t)\chi_e(\mathbf{x}_3, t) + \psi''_1(\mathbf{x}_1, t)\phi_e(\mathbf{x}_2, t)\chi_g(\mathbf{x}_3, t) + \dots \quad (5)$$

where the dots indicate other terms irrelevant for the present purpose (for example the states where electrons 2 and 3 are not disturbed by the interaction). Clearly, we have here an entanglement representing a basic spatial correlator (the equivalent in optics would be an Hanbury Brown and Twiss apparatus). If electron 2 is in the excited state then

electron 3 is in the ground state and reciprocally, this is ‘10’ or ‘01’ information associated with two different bits. In the language of Bohr’s interpretation this would mean that an observer can only detect a particle once since there is only one electron 1. However, here we are not in the Copenhagen instrumentalist framework but in the MWI. Our Universe contains only three electrons and no observer at all. Still, with Everett’s web-function Eq. 5 represents an ontological field and the difference between $|e_2\rangle|g_3\rangle$ and $|g_2\rangle|e_3\rangle$ is completely real in an objective sense (for example those states are orthogonal in the Hilbert space). However, now comes the fundamental issue: since there is no collapse how should we interpret the superposition given by Eq. 5 in MWI? A state like $\psi'_1|g_2\rangle|e_3\rangle + \psi''_1|e_2\rangle|g_3\rangle$ could also equivalently be written $\frac{1}{2}(\psi'_1 + \psi''_1)(|g_2\rangle|e_3\rangle + |e_2\rangle|g_3\rangle) + \frac{1}{2}(\psi'_1 - \psi''_1)(|g_2\rangle|e_3\rangle - |e_2\rangle|g_3\rangle)$. Since there is no observer able to collapse the state this equivalent representation shows that we don’t have yet our macroscopic world where electrons appear either here or there but not at both places. While entanglement allowed us to describe a measuring device in quantum mechanics (i.e., it constitutes an example of the shifty split of the Heisenberg cut) it didn’t apparently remove our problem in the MWI. In other words, the fact that we considered more and more electrons didn’t solve the problem it only propagated it to a larger system thanks to entanglement. Still, it was Everett’s hope that entanglement could somehow solve the measurement issue: how could that be ? Everett’s strategy was to consider larger and larger systems until we can consider a conscious being or at least machines or robots sufficiently sophisticated to have memory sequences of recorded events. The hope was that at a certain scale to be defined the ‘collapse’ (i.e. in the language of the MWI the ‘split’) should occur. This is what we should now analyze. Considering the previous example, i.e., Eq. 5, we see that entanglement will indeed propagate to any other systems in interaction with our detectors. If the detector is emitting a signal going to a larger system able to memorize, i.e., to modify in a rather stable way a chemical, or molecular arrangement, in a mechanical ‘brain’ we can imagine entangled state such as

$$\psi'_1(\mathbf{x}_1, t)|\smile\rangle + \psi''_1(\mathbf{x}_1, t)|\frown\rangle + \dots \quad (6)$$

with obvious notations for the states of the brain. I point out that the ‘ket’ notation for the brain is a compact way of speaking about $\langle X_1, X_2, \dots, X_N|\smile\rangle$ where $N \sim 10^{23}$ is the number of particles in the brain (including its environment, the detectors, and ultimately all the Universe). However the representation chosen is here irrelevant. Furthermore, we have

here obviously $\langle \ddot{\smile} | \smile \rangle = 0$ since the unitarity of the quantum evolution should preserve the orthogonality of the pointer states present in Eq. 5 (i.e. $\langle g_2, e_3 | e_2, g_3 \rangle = 0$). The brain states are of course physical states since with Everett and von Neumann we accept ‘psychophysical parallelism’ [1, 3] that is the functionalist view whereby the mind supervenes on the brain like software on hardware. Everett’s strong belief was that the state of awareness or consciousness of the observer or robot is a new essential ingredient in the theory. However, don’t forget that Everett believed strongly in functionalism and that for him the introduction of minds in quantum mechanics was very different from what was later assumed by believers in the so called many-minds interpretation(s) [9] in which ‘minds’ different of brain states (i.e. without obvious supervenience relations with the hardware) played a fundamental role as well. I will not discuss such alternative approaches here since the abandonment of the psychophysical parallelism is definitely too much for me.

Going back to Eq 6, we see that these two awareness states exist and evolve as if they were alone. The reason is that they constitute two independent solutions of Eq. 1 (I will develop that important point below). But as we said before unitarity allows many representations of Eq. 6 like for example

$$\frac{(\psi'_1 + \psi''_1)}{\sqrt{2}} \frac{(|\smile\rangle + |\ddot{\smile}\rangle)}{\sqrt{2}} + \frac{(\psi'_1 - \psi''_1)}{\sqrt{2}} \frac{(|\smile\rangle - |\ddot{\smile}\rangle)}{\sqrt{2}} + \dots \quad (7)$$

Here we have superposed ‘cat’ states $|\smile\rangle \pm |\ddot{\smile}\rangle$ the meaning of which is unclear. What is the ‘feeling’ of a brain in such a cat state ? Is this not a fatal problem for MWI? Are we not introducing furtively a new axiom favoring a representation, i.e., a preferred basis at the detriment of not preserving full unitarity?

However we point out that for Everett the awareness basis is not so much privileged but better considered as ‘special’ or ‘particular’ in the sense that we are always free to use a different basis if we wish. Therefore, unitarity is not violated. Still, to be convincing, the MWI should explain why $|\smile\rangle \pm |\ddot{\smile}\rangle$ is not an awareness state. May be there are actually such mental states like $|\smile\rangle \pm |\ddot{\smile}\rangle$ but that we can not feel how it is to be like them. Maybe the question is a bit like asking how it feels like to be a bee or a cat. Maybe not. This is a bit magic or mysterious for many and it is probably why some tried to shift from the MWI to the many-minds interpretation(s). Of course, if the two brain states evolve independently there is no apriori reason to mix them but what is more is that we can apparently use an argumentation based on inter-subjectivity and entanglement to see why we never meet

people in such cat states. Indeed, if I met You in the street, going out of the lab, we could create an entangled state like

$$\psi'_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle + \psi''_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle + \dots \quad (8)$$

Therefore, there has to be an inter-subjective agreement between awareness states of Me and You. Moreover, like for Eq. 6 and 7, there are also states like $|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle \pm |\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle$ but now they involve Me, You and all the part of the Universe having interacted with us. There is thus an inter-‘subjective’ agreement between the cat states since Eq. 8 can be written like

$$\begin{aligned} & \frac{(\psi'_1(\mathbf{x}_1, t) + \psi''_1(\mathbf{x}_1, t))}{\sqrt{2}} \frac{(|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle + |\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle)}{\sqrt{2}} \\ & + \frac{(\psi'_1(\mathbf{x}_1, t) - \psi''_1(\mathbf{x}_1, t))}{\sqrt{2}} \frac{(|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle - |\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle)}{\sqrt{2}} \dots \end{aligned} \quad (9)$$

but you should not bother about this since neither Me nor You are feeling such cat states separately.

However, this argumentation is not completely convincing to everyone (see for example Penrose [10]) since there are apparently other counterintuitive ways to write Eq. 8. For instance what about writing Eq. 8 like

$$\begin{aligned} & \frac{(\psi'_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}\rangle + \psi''_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}\rangle)}{\sqrt{2}} \frac{(|\ddot{\smile}_{\text{You}}\rangle + |\ddot{\smile}_{\text{You}}\rangle)}{\sqrt{2}} \\ & + \frac{(\psi'_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}\rangle - \psi''_1(\mathbf{x}_1, t)|\ddot{\smile}_{\text{Me}}\rangle)}{\sqrt{2}} \frac{(|\ddot{\smile}_{\text{You}}\rangle - |\ddot{\smile}_{\text{You}}\rangle)}{\sqrt{2}} + \dots? \end{aligned} \quad (10)$$

The cat state for You are now involved while I am mixed with the first electron state. The well known problem with such an expression is that it is not obvious and univocal to factorize the Me and You like I did (this is the reason why I wrote Me and You inside the same ket vector in Eqs. 8, 9). Indeed, the two protagonists are strongly interacting with a complex environment. The possibility to separate Me from You is therefore in large part arbitrary and in practice physically impossible in the lab. Where to put the border between Me and You is a bit a question of taste. This is clearly reminiscent of the Wigner’s friend paradox: If before interacting with my friend I am in the state $|\ddot{\smile}_{\text{Me}}\rangle + |\ddot{\smile}_{\text{Me}}\rangle$ it is hoped that after meeting we should be in a state like $|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle + |\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle$ and that possible state factorization doesn’t make any sense. Also, for the same reason a state like $|\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle + |\ddot{\smile}_{\text{Me}}, \ddot{\smile}_{\text{You}}\rangle$ can not be easily factorized as $(|\ddot{\smile}_{\text{Me}}\rangle + |\ddot{\smile}_{\text{Me}}\rangle)|\ddot{\smile}_{\text{You}}\rangle$ since the border separating Me from You is fuzzy and shift. We would like to obtain some kind of

superselection rules [11] here in order to prove that such strange cat states are forbidden in nature. For such reasons, many proponents in the MWI like D. Zeh [12] or D. Wallace and S. Saunders [1, 13], following also W. Zurek [13], often emphasize that decoherence should be taken into account in this argumentation.

Decoherence is the averaging of an operator over the ‘irrelevant’ degrees of freedom associated with the environment [12]. The idea is that a macroscopic quantum object like a brain should be considered as an open system interacting with its environment. By averaging over degrees of freedom associated with this environment we can transform a pure system into a mixture characterized by for example rate equations. Overlap terms between different possible environments will decay in time very quickly so that we can use a mixture instead of a pure evolution in the Hilbert space. This is a nice way for removing quantum interferences from a reduced evolution and it is reminiscent of the famous which-path-experiment in the Young double slit setup. Decoherence supposes already that we can interpret $|\psi|^2$ as a probability in agreement with Born’s rule Eq. 2. This creates problems for both Bohr’s and Everett’s interpretations but not for the PWI. Indeed, for Bohr the observer or the apparatus is a key ingredient. But, tracing over some degrees of freedom means that the environment is actually needed as an observer in this theory (‘the environment is watching you’). This is an amendment to the original Copenhagen interpretation but this is not actually so dramatic since the theory is instrumentalist so that improving on it does not really touch the problem of ontology. This problem doesn’t exist at all for the PWI since probabilities have a classical meaning here. Tracing over the environment will only introduce a supplementary emerging ignorance like in thermodynamics. For the MWI however, there is no yet probability so that decoherence cannot be interpreted in the same way. We have only degrees of reality and measure of existence so that tracing actually means summing over some of these degrees of reality. How can this help the MWI? It seems rather to create additional problems. The answer is not so clear because in my opinion proponents of the MWI didn’t yet reach a consensus. One point which is often emphasized is that the privileged basis should be very robust in the sense that only in this basis is the system immune to the interaction with the environment. This means in particular that if an experimentalist was able to prepare and isolate during even a short period a superposition like $|\smiley\rangle \pm |\frowny\rangle$ the system should decohere very fastly, i.e., in a time probably smaller than $10^{-20}s$ [14]. While this result is generally interpreted using probability it has also an absolute meaning as a measure-theoretical way

to define effective orthogonality between independent observer branches.

In this context the ideas of D. Wallace [1, 13, 15] on ‘patterns’ are very interesting for the MWI. What is a pattern? Referring to D. Dennett’s work, Wallace interprets a pattern like a cat or a tiger as an emerging structure inside the quantum evolution. The border of an emerging structure is not always clearly defined at the macroscopical scale since the number of atoms is huge. Including some atoms in You or Me is a bit arbitrary so that we generally don’t care about this shifty border between observers. We already discussed this problem before and we should develop this a bit more. Rigorously, we should call a pattern any complete solution of Eq. 1 for a given Hamiltonian. Referring again to the single electron problem if $|\text{here}(t)\rangle$ and $|\text{there}(t)\rangle$ are separate solutions and therefore patterns then the sum $|\text{here}(t)\rangle + |\text{there}(t)\rangle$ is also representing a viable pattern of the theory. However, the linearity doesn’t destroy the individual patterns ‘here’ and ‘there’ and in that sense it really means that we have two localized electrons here. In the same sense $|\smiley\rangle$ and $|\frowny\rangle$ are representing patterns if they are independent solutions of the full evolution (including the environment). The linear superposition Eq. 6 is also an allowed pattern so that we have really two ‘Me’ here: one happy and one unhappy (Wallace calls that favoring multiplicity over superposition [15]). Moreover, while the orthogonality of the states was here a consequence of the orthogonality of the pointer states this is not necessary. Two solutions of Eq. 1 can be viable patterns without being orthogonal. Still, in practice decoherence will ensure that $\langle \frowny | \smiley \rangle \approx 0$ is a good approximation after a very short time due to the complexity of the environment [11] (neglecting Poincaré’s recurrence over the finite time of human or Universe existence) and we could say that decoherence somehow create and select emerging patterns behaving classically (i.e., without interference) from the full spectrum of possible solutions (those solutions that have no good thermodynamical properties are not considered).

There is an important point to emphasize here: considering a single photon of energy E interacting with a beam splitter, we generally say that we end up with two independent possibilities $|\text{reflected}\rangle$ and $|\text{transmitted}\rangle$ which are added and represent two independent pattern solutions of the free hamiltonian. This is however an approximation since the boundary conditions at the beam splitter make these two ‘solutions’ inseparable. Only the sum $|\text{reflected}\rangle + |\text{transmitted}\rangle$ is a solution of the time independent scattering problem. Still, for practical purposes we are dealing with finite wave packets well localized in space and time. This means that after interacting with the external potential (the beam splitter) we can

approximately and asymptotically consider the two solutions $|\text{reflected}\rangle$ and $|\text{transmitted}\rangle$ as independent and evolving freely. So when we say that we end up with two independent branches this is already an approximation even with very simple systems and without including macroscopic decoherence. I mention that because it is also trivially true for Eqs. 6, and 8 and patterns like $\psi'_1(\mathbf{x}_1, t)|\smile\rangle + \psi''_1(\mathbf{x}_1, t)|\frown\rangle$ or $\psi'_1(\mathbf{x}_1, t)|\smile_{\text{Me}}, \smile_{\text{You}}\rangle + \psi''_1(\mathbf{x}_1, t)|\frown_{\text{Me}}, \frown_{\text{You}}\rangle$ are not actually made of two independent sub-solutions but only constitute complete and inseparable solutions of the full Universal evolution. If you think about that then you realize that indeed $|\smile\rangle$ and $|\frown\rangle$ cannot in general be rigorously independent patterns albeit seen as emerging and approximate structures whose condition $\langle \frown | \smile \rangle \approx 0$ is very robust, due to decoherence. Of course, cat patterns, as given in Eqs. 7 and 9, are not exact solutions as well and we cannot really separate them from the full evolution. This at least shows that the old debates about a preferred basis in the MWI is a bit empty. However, if patterns have only an approximative meaning, they also let the question about the meaning of cat states $|\smile\rangle \pm |\frown\rangle$ unanswered. It is probably better for these reasons to return to our provocative answer: To be a in a state like $|\smile\rangle \pm |\frown\rangle$ is perhaps a bit like feeling as a bee or a cat. Maybe it is here with us during all our life. Maybe not. The ontology of the MWI is definitely very strange and debates about its self-consistency will certainly continue during many years in many-worlds.

III. PROBABILITY IN THE MANY-WORLDS INTERPRETATION

Well, if this is enough for the ontology in the MWI interpretation, what about probability? This is the weaker (worst?) point in the theory and it stirred up so much emotional debates within the years that it could be too long to summarize all points and arguments here. Additionally, since decoherence needs a definition of probability to work, it seems that introducing probability could also help the ontological problem in the MWI. Now, Everett introduced probability in his MWI in two ways. The first way is actually predating Gleason's theorem [16] which Everett discovered independently in a simplified version. I will not summarize this well known reasoning but just remind that its aim is to find the most plausible measure $\mu(\Psi)$ in the Hilbert space which could represent a probability of occurrence. The result, using some natural assumptions about linearity, is that the most natural measure is the one given by Born's law, i.e., Eq. 2. Still, this is not yet a

probability but just the proof that if we are going to attribute a probability to the Ψ state then Born's law is the best choice [1]. In the recent years D. Deutsch [17], D. Wallace and S. Saunders [1, 13] on the one side and W. Zurek on the other side [13, 18] tried to find an alternative mathematical demonstration using some other symmetries (called envariance by Zurek, and decision theory by Deutsch) than those considered by Gleason and Everett. The reasoning is that due to entanglement we can in a natural and mathematical way give a precise statement for Laplace's principle of indifference in the quantum world. The result is of course again Born's law. For me the Gleason version and the envariance demonstration have the same value. They both show that if one is going to introduce probability in the quantum world, and therefore in the MWI, then Eq. 2 is the most natural choice. But still there is no need for probability in the MWI outside from experimental considerations foreign to the theory. At that stage it is interesting to make a remark. Everett [1] and later many authors such as Wallace and Brown [13, 15, 19], or Zurek [18] often claimed that their bayesian definition of probability is no better nor worst than in classical physics so that the situation is the same as for the PWI. Everett for example, wrote [1]: *The situation here is fully analogous to that of classical statistical mechanics, where one puts a measure on trajectories of systems in the phase space by placing a measure on the phase space itself, and then making assertions (such as ergodicity, quasi-ergodicity, etc.) which hold for "almost all" trajectories.* By almost Everett means here a measure-theoretic definition like the one proposed by Lebesgue. However, having a measure is not enough to define a probability. For example, we could use Noether symmetry theorem which shows that Maxwell equations involve a conserved current and interpret this result as a probability. Still, this is not necessary ,i.e., not required by the theory. We need a clear ontological statement for introducing probability in a theory and this involves dynamical elements like particle randomness. In the PWI for example, the conservation law $\nabla \cdot \mathbf{J} + \partial_t \rho = 0$ is clearly not enough to generate the probability interpretation it only gives an indication. As a proof observe that Madelung [20] found simultaneously with de Broglie the guidance formula and the quantum potential. But he interpreted them instead as continuous hydrodynamic fluid equations more in phase with the old Schrodinger interpretation. Modern practitioners of the MWI often think that by comparing their own approaches with the one developed for instance by Gibbs using Liouville's theorem could give a legitimacy to the various concepts of probability they propose. However, Gibbs statistical mechanics is nothing without the

physical interpretation proposed by Boltzmann and Maxwell within the kinetic theory (as Gibbs himself recognized in the introduction to his book [21]). Therefore, we should not try to extract too much from Gleason's theorem, decision theory or envariance.

To introduce anyway probability we should love to the second step of Everett's reasoning and consider a statistical experiment. Suppose for instance a single quantum source emitting photons one by one, all directed on a balanced beam splitter. Each photon is either transmitted or reflected with equal probabilities and the detectors (i.e., avalanche photo-diodes) register singular events in only one of the two path at once. The statistics is of course a simple Bernoulli process and the result, following the weak law of large numbers, is naturally in agreement with Born's rule that the number of photons detected in each detectors are equal on the long run. Now, this is of course a reading which make sense in Bohr's instrumentalist interpretation as well as in the PWI. In the the PWI photons follow trajectories determined by Eq. 3 (or some equivalent [22]) and by the initial conditions in the wave packets (i.e. for example the particle's position at a given time). The theory is very similar to classical mechanics and therefore the PWI contains enough ingredients to introduce statistics in quantum mechanics.

In the MWI this is not the case since the continuous wave or field is all what we have. The wave packet impinging on the beam splitter behaves like a classical Maxwell wave: it is separated into two equal parts but there is no probability in this construction. However, should we be surprised? The MWI is indeed based on the pure unitary Schrodinger dynamics which only accepts the regular solution of Eq. 1. There is no singularity, no fine graining in this theory for generating randomness. It is therefore surprising that Everett in his thesis [1] followed by B. DeWitt and his student N Graham [23] believed that probability could appear in the long run in the MWI. It is of course true that in a Bernoulli process with N repetition of the beam splitter experiment there is $W(n_t) = N!/n_t!(N - n_t)!$ possible branches in which n_t photons are transmitted while $N - n_t$ photons are reflected. Using Stirling formula $N! \approx N^N$ (this is a crude approximation) in the long run limit $N \rightarrow +\infty$ we get $W(\tilde{n}_t) \approx 2^N$ for $\tilde{n}_t/N = 1/2$ which means that the overwhelming majority of the Everett 'branches' will be following Born's law. Still, this has only value in a measure-theoretic sense. We added branches to find a degree of reality equal to $W(n_t)/2^N$, but nowhere there is a reason why we should do that. In the same way that typicality is not probability there is here no possibility for extracting 'a tend to from a does'[17]. Additionally, for practical

experiments we don't use an infinite number of registered events. The PWI can deal with that only because the theory is deterministic and because randomness only results from the choice of the initial conditions for the particle positions. There is no need for an infinite number of events in the same way that in kinetic theory a gas doesn't contain an infinite number of molecules (the Gibbs ensemble is just an idealization after all).

I stress that my comments are somehow well known [24] and that we can still find some proponents of the MWI who are convinced that Everett's answer is the good one. In the recent year D. Wallace and S. Saunders following the ideas of D. Deutsch [1, 13] introduced bayesianism into the MWI in order to save the theory (this is what they called decision theory following L. J. Savage [25]). Quoting D. Wallace on his discussion on patterns [15]: *'we have at least shown that it is rational for the observer to assign some weighting: in other words, we have shown that there is room for probabilistic concepts (at least the decision-theoretic sort) to be accommodated in the theory.* In other words, the observer doesn't know where he will go after 'branching' so that it seems legitimate to call that a bayesian choice. Or is it not? Bayesianism following Ramsey or de Finetti is mainly a subjective interpretation of probability based on inferences and degrees of belief as used by poker players or insurance companies. Still, a poker player can only use his subjective notion of probability in connection with empirical evidences. Such empirical evidence means frequency of occurrences (I.e., ideally using an infinite sequence following von Mises cf. however our remarks made earlier concerning Gibbs's ensembles [26]). Therefore we are sent back to our first criticism concerning the Everett, DeWitt-Graham reasoning.

As we explained before the unitary Schrodinger evolution is too regular and simple for implying objective probability and randomness. And it is therefore difficult for me to understand how some could even hope to obtain physical probability directly from subjective bayesianism (my criticism concerns also the new trends about Qbism even though Qbism is actually the Copenhagen counterpart of the decision theoretic view on the MWI). Furthermore, I am feeling that the whole game of decision theory is a bit like meeting a pair of twins or clones (see the very suggestive 'Zaxtarian' scenario proposed by B. Green [27] based on the 'sleeping beauty' analogy by L. Vaidman [28]) and asking one of them what was the probability to be the other. In that case, at least, ontogenesis (the fine grained structures) make some difference but with the quantum states the symmetry is too high and the fine-graining (unlike in the PWI) absent. Why therefore should we introduce probabil-

ity? This is not convincing, and the problem is not that the observer doesn't know where he will go but rather that he will actually choose both ways since he will soon be sliced into two quantum clones. The MWI is deterministic and there is no hidden variable with unknown initial conditions which could tell you what was the path you really followed in a quantum interferometer. Therefore, how can we speak of some results being likely and others being unlikely when both take place? In one sense the strong symmetry required by Zurek's envariance, i.e., the quantum version of Laplace's indifference principle, already ruins the chance of success of the MWI in justifying the use of bayesian probability. Over the years L. Vaidman often emphasized (as a kind of last chance for the MWI?) that probability should be postulated in the MWI [8]. If this is true, then we completely denature the dream of Everett and therefore we will at the end obtain something like a new version of the PWI and return to the original de Broglie's proposal. In the following I will describe some alternatives to the old MWI, which indeed, are going to explore this analogy with the PWI.

IV. SAVING THE MANY-WORLDS?

In the recent years, several approaches have been proposed to modify or complete the MWI. I already mentioned the various many-minds interpretations [9]. Such approaches include in the theory some new ingredients which look a bit like hidden variables in the PWI (in some versions the 'minds' are indeed flying over the wave-function like the electron is surfing the guiding wave in the PWI: this is a romantic view). I will not analyze here such provocative ideas, but the point is, indeed, that one way of saving the MWI is to go in the direction of the PWI. This is what is interesting to me here since the proposal is actually going far beyond the realm of the many minds interpretation(s). Indeed, the analogy with the PWI is for example the path proposed by A. Valentini [13], in his many Bohmian worlds theory. Such an approach is also advocated by several authors like F.J. Tipler and K. J. B  lstrom [29]. The main point is that the PWI allows a description of the wave function of the whole Universe by adding a Gibbsian ensemble of Universes to the theory. Indeed, if we consider an infinite number of copies of such a Universe all characterized by the same wave function Ψ but in which the trajectories $X_\Psi(t)$, for the large vector associated with the N particles of the Universe in the configuration space, differ only by the choices concerning

the initial conditions $X_\Psi(t_0)$, then we have a theory where many Bohmian worlds exist independently of each others. This reminds us of the old criticism by D. Deutsch about PWI as a MWI in a permanent state of chronic denial. Here, the aim is clearly to satisfy both PWI and MWI proponents by reversing the critics. By introducing many copies of the Universe we could, maybe, understand how the quantum potential can depend on the wave function itself. N. Rosen [30] who was one of the first to study the PWI (even before Bohm) gave up on this theory because it was not acceptable for him that a quantum potential Q_ψ should depend somehow on the probability density (remember that for a single non relativistic particle we have $Q_\psi = -\frac{\hbar^2}{2m}\Delta|\psi(x)|/|\psi(x)|$). De Broglie renounced his PWI in 1930 in part for similar reasons: if the wave function is a ‘subjective’ element (the word is from de Broglie), then how could a dynamics depends on statistics? De Broglie came back to PWI in 1952 since this problem was not anymore a serious one for him. Still, we can motivate the many Bohmian worlds view by introducing the quantum potential as a kind of interaction between the different worlds. Actually this is even the only justification for the many Bohmian worlds view. Personally, I am not so much convinced but the theory is fine to study anyway since we don’t know yet where it will lead us.

To conclude this chapter, I would like to discuss a different provocative proposals for modifying the MWI which I imagined some years ago. Since I am not taking it too seriously I will call that model the jumper interpretation (JI). In the JI the idea is to find a stochastic approach based on the MWI. I will consider first a single electron described by $\Psi(x, t)$. My suggestion is that at any time there is a single particle in the wave like in the PWI. The particle is located at x at time t . The density of probability that it will be found at x' at any other time is simply given by

$$P_\psi(x', t') = |\Psi(x', t')|^2. \quad (11)$$

That’s more or less all. The evolution is completely stochastic and if the wave is going to be diffracted then the electron is jumping from one place to a different one simply using Born’s rule. As inPWI, there is an obvious privileged basis here: the spatial coordinate representation but we could choose a different one. Also, the dynamics can look ‘crazy’. First, the electron at time t' has a non vanishing probability to appear at any place in the wave whatever the distance separating the two points x and x' (the velocity $(x - x')/(t - t')$ can therefore diverge). Second, consider a particle interacting with a beam-splitter. After

crossing the apparatus we have now two separated wave packets. Still, from my JI the particle can jump from one beam to the other completely randomly and this even if there is a huge distance between them and even if there is a thick wall as an obstacle. The theory also requires a privileged Lorentz Frame (like PWI). Indeed since there is not limit for velocity the trajectories are allowed to go backward in time in some reference frames. A privileged slicing of space-time allows us to define unambiguously the possible probability of reaching a point using Born's rule $P_\psi(x', t') = |\Psi(x', t')|^2$ calculated in this preferred frame. Actually, this model is a minimalist version of the PWI. Instead of being deterministic it is stochastic. But still it shares with the PWI many common features. In particular, in a state like $|\text{here}(t)\rangle + |\text{there}(t)\rangle$ the particle is in one place at a time if we choose to favor the x representation. If we take as preferred basis the momentum representation then the system will not have position at all but rather a jumping momentum k in the distribution $|\langle k|\text{here}(t)\rangle + \langle k|\text{there}(t)\rangle|^2$. This JI could also be used to explain the EPR paradox by including entanglement and by generalizing it to any possible Hilbert space (for example to include spin or polarization variables). Consider for example a perfectly entangled photon pair state like

$$|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2 \quad (12)$$

(where H, V denote horizontal and vertical polarization for photons 1 and 2). We require a preferred basis for describing the stochastic dynamics, let say, $|H\rangle_i$ and $|V\rangle_i$ (but the choice could be different from pair to pair). When the pair of photons is produced by the source it jumps randomly from time to time into the states $|H\rangle_1|H\rangle_2$ or $|V\rangle_1|V\rangle_2$. Now the EPR pair is directed in the Bell experiment with polarizing beam splitters and wave plates (for selecting polarization bases). Due to the presence at each time of both waves $|H\rangle_1|H\rangle_2$ and $|V\rangle_1|V\rangle_2$ the outcomes will follow the quantum predictions. The situation will be like for the PWI involving a non locality, this time not due to the quantum potential but to the mere existence of the two independent branches $|H\rangle_1|H\rangle_2$ and $|V\rangle_1|V\rangle_2$: one full, one empty at each time.

The JI can be easily extended to measurement situations like the one described by Eq. 5 with many electrons. Since we have privileged some ket basis in this theory the quantum state has a non ambiguous meaning in such a basis. This is again very similar to the PWI. Here however the choice of the privileged basis is arbitrary: I could have chosen a momentum space instead of a spatial coordinate this is my free ontological assumption. Now, of course

there is this strange randomness induced by the jump of particles in the wave function. What is this jump going to imply for the brain states and to the observers? Are they going also to jump or to see these electrons jumping? If the electrons are jumping in front of my eyes then the theory is of course invalidated. But, this is again a hidden variable model and actually the nice thing is that this jump is really hidden (in the same way that Bohmian trajectories are hidden in the PWI). Indeed, we see first that if we consider an awareness state, like $\langle X_1, \dots, X_N | \smile_t \rangle$ written in the preferred basis (the question of the basis choice is again arbitrary but to fix it let's say that I am working mainly with spatial coordinates + the spin degrees of freedom) then the mind somehow supervenes on the N particles position at a given time (this is also true for the PWI). Second, in a state like Eq. 6

$$\Psi(\mathbf{x}_1, X_1, \dots, X_N, t) = \psi'_1(\mathbf{x}_1, t) \langle X_1, \dots, X_N | \smile_t \rangle + \psi''_1(\mathbf{x}_1, t) \langle X_1, \dots, X_N | \frown_t \rangle + \dots \quad (13)$$

there is only one electron 1 at \mathbf{x}_1 and one brain (whatever that means) located at X_1, \dots, X_N . Like for the PWI it doesn't matter if we use Eq. 6 with awareness states or Eq. 7 with cat states. In both the probability of presence is given by the same number $P_\Psi(\mathbf{x}_1, X_1, \dots, X_N, t) = |\Psi(\mathbf{x}_1, X_1, \dots, X_N, t)|^2$. Now, the question about patterns plays an important role here and discussing it now will also bring some new insights to the PWI. What is important, indeed, is that in both the present JI, and in the PWI, the whole system only occupies one position $X = [\mathbf{x}_1, X_1, \dots, X_N]$ at any time, only restricted by the condition $P_\Psi(\mathbf{x}_1, X_1, \dots, X_N, t) \neq 0$. Now, you should remember that we are speaking about a quantum measurement and that $\psi'_1(\mathbf{x}_1, t)$ and $\psi''_1(\mathbf{x}_1, t)$ are not spatially overlapping (at least at some time t). It means that we have either

$$P_\Psi(\mathbf{x}_1, X_1, \dots, X_N, t) = |\psi'_1(\mathbf{x}_1, t)|^2 |\langle X_1, \dots, X_N | \smile_t \rangle|^2 \quad (14)$$

or

$$P_\Psi(\mathbf{x}_1, X_1, \dots, X_N, t) = |\psi''_1(\mathbf{x}_1, t)|^2 |\langle X_1, \dots, X_N | \frown_t \rangle|^2 \quad (15)$$

depending on whether the electron position \mathbf{x}_1 is located in the support of ψ'_1 or ψ''_1 . The functionalist theory together with the existence of particles therefore imposes that we can only be in one of the two awareness states! This trick is not possible in the old MWI interpretation since there is no preferred basis in Eq. 1. Additionally, it apriori gives a reason to impose the spatial coordinates as preferred basis since using momentum with its

delocalized quantum states $\tilde{\psi}'_1(\mathbf{k}), \tilde{\psi}''_1(\mathbf{k})$ could not lead to such a clean resolution of the observer identity crisis. This is, I think, a very good reason to prefer the PWI over the MWI.

Now, with the present stochastic model the state given by Eq. 5 represents an electron 1 and the detectors as jumping together. The three entangled electrons will jump from $\psi'_1|g_2\rangle|e_3\rangle$ to $\psi''_1|e_2\rangle|g_3\rangle$ randomly between two arbitrary times. In the state given by Eq. 6 with an observer entangled with electron 1 the whole system should also jump but, again, only together. That means, that the observer cannot feel this jump at all! If he is jumping all his memory sequence and the environment is jumping with him (this why I called the theory crazy). Furthermore, if other observers are included in the formalism (see Eq. 8) then the subjective agreement will also be automatic in the appropriate basis and You will jump with Me always in agreement with quantum mechanics. I emphasize that this JI is certainly a schematic structure not to be taken too seriously; but I think that it contains mainly all the ingredients for a good hidden variable theory. In particular, the concept of probability used in the model is clear enough to bypass all the contradictions contained in the MWI. As a further development we could also present a many-jumpers interpretation similar in philosophy to the many bohmian worlds approach discussed before. Again, this could bring some philosophical advantages for explaining how the particles ‘know’ where to jump. The inter communication between the different jumping worlds brings indeed such an information to the whole ensemble of Universes and with this we will conclude our ‘many-stories’ theory.

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- [1] H. Everett, Relative state formulation of quantum mechanics, *Reviews of Modern Physics* **29** 454-462 (1957); J. Barrett, P. Byrne, H. Everett, *The Everett interpretation of quantum mechanics - Collected Works 1955-1980 with Commentary*, Princeton University Press (2012); D. Wallace, *The emergent multiverse: quantum theory according to the Everett Interpretation*, Oxford University Press (2012).

- [2] P. R. Holland, *The Quantum Theory of Motion*, Cambridge University Press, Cambridge, 1993. See also of course: D. Bohm, 'A suggested interpretation of the quantum theory in terms of "hidden" variables. ' Part I, *Phys. Rev.* **85**, 166 (1952); Part II, **85** 180 (1952). The fundamental work by de Broglie is discussed in L. de Broglie, 'La mécanique ondulatoire et la structure atomique de la matière et du rayonnement', *J. Phys. Radium* **8**, 225 (1927) and in the report of the 5th Solvay's conference (in french) reproduced recently in: G. Bacciagaluppi and A. Valentini (ed.s): *Quantum Theory at the Crossroads - Reconsidering the 1927 Solvay Conference*, Cambridge University Press (2007). The best book on the subject is still probably: D. Bohm and B. J. Hiley, *The Undivided Universe - An Ontological Interpretation of Quantum Theory*, Routledge, London (1993).
- [3] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, *Grundlehren der mathematischen Wissenschaften*, Bd. 38., Springer, Berlin, 1932.
- [4] J. S. Bell, On the problem of hidden variables in quantum mechanics , *Rev. Mod. Phys.* **38** , 447 (1966).
- [5] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete? ". *Physical Review* **47**, 777 (1935).
- [6] J. S. Bell, On the Einstein Podolsky Rosen Paradox, *Physics* **1**, 195-200 (1964).
- [7] C. Callender, The emergence and interpretation of probability in Bohmian mechanics, *Studies in History and Philosophy of Modern Physics* **38**, 351-370 (2007).
- [8] L. Vaidman, Quantum theory and determinism, *Quantum Studies: Mathematics and Foundations* **1**, 5-38 (2014); L. Vaidman, Many-Worlds Interpretation of Quantum Mechanics, *Stan. Enc. Phil.*, E. N. Zalta (ed.) (2014), [<http://plato.stanford.edu/entries/qm-manyworlds/>]; L. Vaidman, Probability and the many-worlds Interpretation of quantum theory, in *Probability in Physics*, *The Frontiers Collection*, Y. Ben-Menahem and M. Hemmo (eds.), pp. 299-311, Berlin Heidelberg, Springer-Verlag [[arXiv:quant-ph/0111072](https://arxiv.org/abs/quant-ph/0111072)] (2012); L. Vaidman, Probability in Physics, *The Frontiers Collection*, Y. Ben-Menahem and M. Hemmo (eds.), pp. 299-311, Berlin Heidelberg, Springer-Verlag (2012). 0706.1348; L. Vaidman, On schizophrenic experiences of the neutron or why we should believe in the many-worlds interpretation of quantum theory, *International Studies in the Philosophy of Science* **12**, 245-261 (1998); L. Vaidman, The measure of existence of a quantum world and the Sleeping Beauty Problem, *Analysis* **73**, 695-706 (2013).

- [9] D. Albert and B. Loewer, Interpreting the many worlds interpretation, *Synthese* **77**, 195-213 (1988); M. Lockwood, *Mind, brain and the quantum*, Oxford: Basil Blackwell (1989).
- [10] R. Penrose, *The large, the small and the human mind*, Cambridge University Press (2000).
- [11] E. Joos, Elements of environmental decoherence, in *Decoherence: theoretical, experimental, and conceptual problems*, edited by P. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.-O. Stamatescu, Springer (1999).
- [12] M. Schlosshauer, *Decoherence and the quantum-to-Classical transition*, Heidelberg and Berlin: Springer (2007); E. Joos, H.D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.-O. Stamatescu: *Decoherence and the appearance of a classical world in quantum theory*. 2nd edition (Springer, Berlin 2003).; H. D. Zeh, *The physical basis of the direction of time*, Springer (2007); H. D. Zeh, *The Problem of conscious observation in quantum mechanical description*, *Epistemological Letters* **63** (1981) [arXiv:quant-ph/9908084v3].
- [13] *Many Worlds? Everett, quantum theory, and reality*, Edited by S. Saunders, J. Barrett, A. Kent, and D. Wallace, Oxford University Press (2010).
- [14] M. Tegmark, The importance of quantum decoherence in brain processes, *Phys. Rev. E* **61**, 4194 (2000).
- [15] D. Wallace, Everett and structure, *Studies in the History and Philosophy of Modern Physics* **34**, 87-105 (2003).
- [16] A. M. Gleason, Measures on the closed subspaces of Hilbert space, *Journal of Mathematics and Mechanics* **6**, 885-894 (1957).
- [17] D. Deutsch, Quantum theory of probability and decisions. *Proceedings of the Royal Society of London A* **455**, 3129-3137 (1999).
- [18] W. Zurek, Decoherence, einselection, and the quantum origins of the classical, *Review of Modern Physics* **75**, 715-775 (2003).
- [19] H.R. Brown, D. Wallace, Solving the measurement problem: de Broglie-Bohm loses out to Everett, *Found. Phys.* **35**, 517-540 (2005).
- [20] E. Madelung, Quantentheorie in hydrodynamischer Form, *Zeit. F. Phys.* **40**, 322-326 (1927).
- [21] J. W. Gibbs, *Elementary principles in statistical mechanics*, developed with special reference to the rational foundation of thermodynamics, New York, Dover Publications (1960).
- [22] A. Drezet, Comment on “A simple experiment to test Bell’s inequality”, J.-M. Vigoureux, *Opt. Commun.* **250**, 370 (2005).

- [23] B. DeWitt, Quantum mechanics and reality, *Physics Today*, **23**, 30-35 (1970; N. Graham, The Measurement of relative frequency, in DeWitt and N. Graham (eds.) *The Many-Worlds Interpretation of Quantum Mechanics*, Princeton NJ, Princeton University Press (1973).
- [24] L. E. Ballentine, Can the statistical postulate of quantum theory be derived? a critique of the many-universes interpretation, *Found. Phys.* **3**, 229-240 (1973); P. Benioff, A note on the Everett interpretation of quantum mechanics, *Found. Phys.* **8**, 709-720 (1977); E. J. Squires, On an alleged ‘proof’ of the quantum probability law, *Phys. Lett. A* **145**, 67-68 (1990).
- [25] J.L. Savage, *The Foundations of statistics*, Wiley (1954).
- [26] D. Wallace often criticized the frequentist view using the well known redundancy with the weak law of large numbers. Indeed, this weak law is sometime used, improperly, as a proof of the long run probability formula. However, since the weak law already needs probability (with its frequentist definition) to run there is obviously a kind of circularity and this motivates Wallace’s bayesian reading of probability. I however think that the naive view that the weak law is a proof of something like the frequentist interpretation is not exact. The frequentist view, indeed, is not proven by the weak law of large number: it only shows that this view is consistent with itself. Actually, the frequentist view as often taught is too simple for exhausting the full physical complexity of randomness. As I mentioned, Gibbs ensembles are pure idealization and what we need is actually a physical postulate concerning the initial conditions of the Universe. If these conditions impose that over $N \approx 10^{23}$ molecules N_1 will be in state 1 and $N_2 = N - N_1$ in state 2 then we have already a good basis for probability. However, this is not enough in order to have a good random Universe since in the long run a maverick result like 00...11 (with N_1 ‘0’ arriving first and N_2 ‘1’ after) is not random but would however satisfy the naive frequentist view. In my opinion, we must also add a physical postulate similar to Boltzmann’s stosszahlansatz telling that the repartition of ‘0’ and ‘1’ in the finite sequence is somehow ‘random’ as determined by initial conditions. This postulate would for example mean that the molecules with state 1 and 2 are homogeneously distributed over the box or the Universe considered, and that the observer should pick-up a fair sample when making a statistics. I agree that mathematicians will not like my postulates, but in my opinion probability should be a physical science before being a domain for mathematicians. In other words, probabilities are only pure idealizations in a deterministic Universe and they only approximately convey a language for what we call chance and randomness. Unfortunately we don’t have a better one.

- [27] B. Green, The hidden reality: parallel universes and the ddp laws of the cosmos, New York, Vintage Books (2011).
- [28] L. Vaidman, Probability and the many worlds interpretation of quantum theory. In Quantum theory: Reconsideration of foundations, ed. A. Khrennikov, Sweden, Vaxjo University Press, 408-412. (2001).
- [29] F. J. Tipler, What about quantum theory? Bayes and the Born interpretation, [arXiv:quant-ph/0611245] (2006); K.J. Boström, Combining Bohm and Everett: Axiomatics for a standalone quantum mechanics, [arXiv:1208.5632] (2012); M.J.W. Hall, D.-A. Deckert, H.M. Wiseman, Quantum phenomena modelled by interactions between many classical worlds, Phys. Rev. X **4**, 041013 (2014); C.T. Sebens, Quantum mechanics as classical physics, [arXiv:1403.0014] (2014).
- [30] N. Rosen, On waves and particles, Journal of the Elisha Mitchell Scientific Society **61**, 67-73 (1945).